

Arbitrary Order Constant Curvature Midplane Magnetic Field Expansion

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- Start with vertical magnetic field in the midplane
 - ◆ Horizontal and longitudinal fields are zero in midplane
- Reference orbit is planar
- Curvature is constant
- None of these are necessary; they simplify the computation

- Write vector potentials as a power series in y (α is the direction, x , y , or s)

$$A_{\alpha}(x, y, s) = \sum_{k=0}^{\infty} \frac{1}{k!} A_{\alpha k}(x, s) y^k$$

- Find expressions for these coefficients

$$A_{x,2k} = \sum_{n=0}^{k-1} \left[a_{xkn} A_{k,2n+1}(x, s) + \sum_{m=0}^{2k-2n-3} b_{xknm} B_{k,2n+1,m}(x, s) \right]$$

$$A_{s,2k} = \sum_{n=0}^k \left[a_{skn} A_{k,2n+1}(x, s) + \sum_{m=0}^{2k-2n-1} b_{sknm} B_{k,2n,m}(x, s) \right]$$

$$A_{kj}(x, s) = \frac{h^{2k-j}}{(1+hx)^{2k+1}} \int_0^x (1+h\bar{x}) \partial_s^j B_y(\bar{x}, 0, s) d\bar{x}$$

$$B_{kjm}(x, s) = \frac{h^{2k-j-m-1}}{(1+hx)^{2k-m-1}} \partial_x^m \partial_s^j B_y(x, 0, s)$$

- Relatively simple expressions
- There are recursion relations for the coefficients, starting with $a_{s00} = 1$; the coefficients are (large) integers
- Starting with B_x and B_s nonzero will only double the amount of work
 - ◆ Does midplane expansion make sense in this case? Maybe if small.
- Can also compute A_y and components of B similarly.
 - ◆ B components will not involve $A_{kj}(x, s)$
- Future work
 - ◆ s variation in h
 - ◆ Nonzero h_y
 - ◆ Multipole expansion
 - ◆ Not clear how nice these will come out